Chapter 6. **EINSTEIN'S THEORY of RELATIVITY**

In this chapter we consider Einstein's Physical Special Theory of Relativity. However, we will not be dealing with the 4-dimensional Einstein space, (t, x, y, z), but instead, we will be dealing with the 2-dimensional one, (t, x). Hence, we can consider two coordinate systems S and S', where $y = y'$, $z = z'$, and movement along the $x-$axis. Let c be an independent constant, so that we have movement from $M(t, x)$ to $\tilde{M}(\tilde{t}, \tilde{x})$ in S and, similarly, from $M(t', x')$ to $\tilde{M}(\tilde{t}', \tilde{x}')$ in S' with speed c. Now, assume $\tilde{x} \geq x$, $\tilde{t} \geq t$, $\tilde{x}' \geq x'$, and $\tilde{t}' \geq t'$, so then we have $\tilde{x} - x = c(\tilde{t} - t)$ and $\tilde{x}' - x' = c(\tilde{t}' - t')$, i.e. $\tilde{x} - x - c(\tilde{t} - t) = 0$ and $\tilde{x}' - x' - c(\tilde{t}' - t') = 0$. Now, by Einstein, we shall have $\tilde{x} - x - c(\tilde{t} - t) \equiv \tilde{x}' - x' - c(\tilde{t}' - t')$. Consider now the standard coordinate change $\begin{cases} x' = a_1 x + b_1 t \\ t' = a_2 x + b_2 t \end{cases}$ so then $\begin{cases} \tilde{x}' = a_1 \tilde{x} + b_1 \tilde{t} \\ \tilde{t}' = a_2 \tilde{x} + b_2 \tilde{t} \end{cases}$. Now, assume we are in W_{30}, then we have

$$
\begin{aligned}
&\tilde{x} -_{30} x -_{30} c \times_{30} (\tilde{t} -_{30} t) \equiv \\
&\equiv a_1 \times_{30} \tilde{x} +_{30} b_1 \times_{30} \tilde{t} -_{30} a_1 \times_{30} x -_{30} b_1 \times_{30} t - \\
&-_{30} c \times_{30} (a_2 \times_{30} \tilde{x} +_{30} b_2 \times_{30} \tilde{t} -_{30} a_2 \times_{30} x -_{30} b_2 \times_{30} t)
\end{aligned}
$$

and $c = 3 \cdot 10^8$ meters/sec.

Now, we can solve for $(\Delta x, \Delta t)$ the following equation:

$\Delta x -_{30} a_1 \times_{30} \Delta x +_{30} 3 \cdot 10^8 \times_{30} (a_2 \times_{30} \Delta x) = 3 \cdot 10^8 \times_{30} \Delta t -_{30} 3 \cdot 10^8 \times_{30} (b_2 \times_{30} \Delta t) +_{30} b_1 \times_{30} \Delta t$. In order to do that, we have to satisfy the following assumptions: the addition and multiplication has to be done in the order as it is written, respecting parentheses, all intermediate results must be in W_{30}. The solution procedure will occur as follows: we will give Δx and will try to find Δt by brute force, so our solution will either be $(\Delta x, \Delta t)$ or $(\Delta x, \Delta t \text{ d.n.e.})$. To begin with, we take $a_1 = b_2 = 1.000000000000000138888888888888$, $b_1 = -5.000000000000000694444444444444$ and $a_2 = -0.000000000000000055555555555555$. Then $\Delta x = 10^{-30}$ and $\Delta t = 0$ turns out to be the solution of the general equation. What this means, is that small displacements can occur instantaneously, which contradicts Einstein's conclusions.

Generally, we have a problem of determining existence of Δt for a given Δx. Hence we can only talk about solutions to questions such as "given Δt, find Δx." Here is an illustration of this fact: suppose $\Delta x = 5$. Then

$$\begin{aligned}
&5 -_{30} 3\cdot 10^8 \times_{30} \Delta t = \\
&= a_1 \times_{30} 6 +_{30} b_1 \times_{30} (1 +_{30} \Delta t) -_{30} (a_1 +_{30} b_1) - \\
&-_{30} 3\cdot 10^8 \times_{30} (a_2 \times_{30} 6 +_{30} a_1 \times_{30} (1 +_{30} \Delta t) -_{30} (a_2 +_{30} a_1))
\end{aligned}$$

and hence we have the following equation:

$$-3\cdot 10^8 \times_{30} \Delta t +_{30} 3\cdot 10^8 \times_{30} (a_1 \times_{30} \Delta t) +_{30} b_1 \times_{30} \Delta t = 0.00000008333333402777694444444 .$$

Now, assume we have $x_1 \neq x_2$ and $t_1 = t_2 = t$ and find t_1', t_2' in S'. We have

$\begin{cases} t_1' = a_2 \times_{30} x_1 +_{30} b_2 \times_{30} t \\ t_2' = a_2 \times_{30} x_2 +_{30} b_2 \times_{30} t \end{cases}$ and hence $t_2' -_{30} t_1' = a_2 \times_{30} x_2 -_{30} a_2 \times_{30} x_1$. Now, if $x_2 = x_1 +_{30} 10^{-14}$ and $x_1 = 1$ so that $x_2 = 1 +_{30} 10^{-14}$, then $a_2 \times_{30} x_2 = a_2$ and $a_2 \times_{30} x_1 = a_2$ and hence $t_2' - t_1' = 0$. Therefore, the two events take place simultaneously, again, contradicting Einstein, however, if we take $x_2 - x_1 = 10^{-13}$ (or larger), then $t_2' - t_1' \neq 0$ and the new results coincides with Einstein.

Next phenomenon we consider is the delay of a stopwatch, which in our case does not occur for small parameter values, as opposed to Einstein. Suppose that in S' we have a fixed stopwatch (so that x' is a constant), which counts time t'. We will look at what the stopwatch is showing in S according to the map $t_1 \to t_1'$. We have $\begin{cases} t_1' = a_2 \times_{30} x_1 +_{30} b_2 \times_{30} t_1 \\ x_1' = a_1 \times_{30} x_1 +_{30} b_1 \times_{30} t_1 \end{cases}$ and

$\begin{cases} t_2' = a_2 \times_{30} x_2 +_{30} b_2 \times_{30} t_2 \\ x_2' = a_1 \times_{30} x_2 +_{30} b_1 \times_{30} t_2 \end{cases}$, then $x_1' = x_2'$ implies that

$\begin{cases} a_1 \times_{30} x_1 +_{30} b_1 \times_{30} t_1 = a_1 \times_{30} x_2 +_{30} b_1 \times_{30} t_2 \\ t_2' - t_1' = a_2 \times_{30} x_2 +_{30} b_2 \times_{30} t_2 -_{30} a_2 \times_{30} x_1 -_{30} b_2 \times_{30} t_1 \end{cases}$ and hence

$\begin{cases} a_1 \times_{30} x_2 -_{30} a_1 \times_{30} x_1 = b_1 \times_{30} t_1 -_{30} b_1 \times_{30} t_2 \\ t_2' - t_1' = a_2 \times_{30} x_2 -_{30} a_2 \times_{30} x_1 +_{30} b_2 \times_{30} t_2 -_{30} b_2 \times_{30} t_1 \end{cases}$. Now, let $t_2' - t_1' = 10^{-30}$. We are looking for $t_2 - t_1 = 10^{-30}\left(= t_2' - t_1'\right)$. Let $x_1 = 1$, $x_2 = 1 +_{30} 5 \cdot 10^{-30}$, $t_1 = 1$, and $t_2 = 1 +_{30} 10^{-30}$, then

$\begin{cases} a_1 \times_{30} x_2 -_{30} a_1 \times_{30} x_1 = a_1 +_{30} 5 \cdot 10^{-30} -_{30} a_1 = 5 \cdot 10^{-30} \\ b_1 \times_{30} 1 -_{30} b_1 \times_{30} \left(1 +_{30} 10^{-30}\right) = b_1 -_{30} b_1 +_{30} 5 \cdot 10^{-30} = 5 \cdot 10^{-30} \end{cases}$, i.e. after we plug in corresponding values into both equations, we see that equalities hold, therefore $t_2' - t_1' = t_2 - t_1$ and the stopwatch does not change, contradicting Einstein. However, whenever $t_2' - t_1' \geq 10^{-14}$, the stopwatch does change, agreeing with Einstein.

Similarly, we can consider the reduction in length of a rod when traveling at light speed. In our case, this actually does not occur whenever the parameters are small enough. Let there be a rod of length l' at time t', so $x_2' - x_1' = l'$. Now, we have x_1 and x_2 at time t in S, so we have $\begin{cases} x_1' = a_1 \times_{30} x_1 +_{30} b_1 \times_{30} t \\ x_2' = a_1 \times_{30} x_2 +_{30} b_1 \times_{30} t \end{cases}$ with $l' = x_2' - x_1' = a_1 \times_{30} x_2 -_{30} a_1 \times_{30} x_1$ and we need to connect l' and $l = x_2 - x_1$. Let a_1 be as above. Now, let for instance, $l' = 10^{-30}$ and also let $x_1 = 1$, and $x_2 = 1 +_{30} 10^{-30}$. Then $a_1 \times_{30} x_1 = a_1$ and $a_1 \times_{30} x_2 = a_1 +_{30} 10^{-30}$, therefore, $l = l'$. Similarly, we can show that $l = l'$ whenever $l' = 10^{-15}$, contradicting Einstein. However, whenever $l' \geq 10^{-14}$ there will be no solution, since $a_1 \times_{30} x_1 = a_1$ and $a_1 \times_{30} x_2 = a_1 +_{30} 10^{-14} +_{30} 10^{-30}$, hence there is agreement with Einstein.