# *Analogy of Fermat's Last Problem in Observer's Mathematics -*

# *- Mathematics of Relativity*

Boris Khots

Dmitriy Khots

bkhots@mchsi.com

dkhots@mchsi.com

# Background

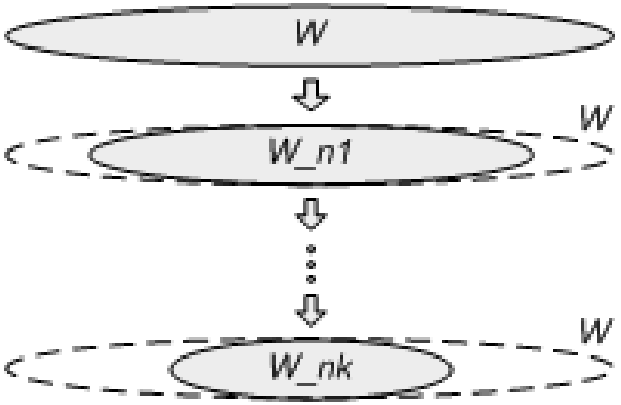
- $W$ – set of all real numbers.
- $W_n$ – set of all finite decimal fractions of length $2n$.
- $W_n = \{\underbrace{\star\cdots\star}_{n}.\underbrace{\star\cdots\star}_{n}\}$.
- Concept of *observers*.

# Observers

- All observers are naive.
- Each *thinks* that he lives in $W$, but
- Each *deals* with $W_n$, so called $W_n$-observer.
- Each sees more naive observers, i.e.,
- $W_{n_1}$-observer can identify that $W_{n_2}$-observer is naive if $n_1 > n_2$.

# Observers - More Specifically

- Assume $n_1 > n_2$, then
- $\star \to \infty$ for $W_{n_2}$-observer means $\star \to 10^{n_2}$ for $W_{n_1}$-observer.
- $\star \to 0$ for $W_{n_2}$-observer means $\star \to 10^{-n_2}$ for $W_{n_1}$-observer.
- For $n_1 > n_2 > \cdots > n_k$, visual example:



# Arithmetic - Addition & Subtraction

- For $c = c_0.c_1...c_n$, $d = d_0.d_1...d_n \in W_n$

$$c \pm_n d = \begin{cases} c \pm d, \text{ if } c \pm d \in W_n \\ \text{not defined, if } c \pm d \notin W_n \end{cases}$$

write $((... (c_1 +_n c_2) ...) +_n c_N) = \sum_{i=1}^{N} {}^n c_i$ for $c_1, ..., c_N$ iff the contents of any parenthesis are in $W_n$.

# Arithmetic - Multiplication

- For $c = c_0.c_1...c_n$, $d = d_0.d_1...d_n \in W_n$

$$c \times_n d = \sum_{k=0}^{n}{}^{n} \sum_{m=0}^{n-k}{}^{n} 0.\underbrace{0...0}_{k-1} c_k \cdot 0.\underbrace{0...0}_{m-1} d_m$$

where $c, d \geq 0$, $c_0 \cdot d_0 \in W_n$, $0.\underbrace{0...0}_{k-1} c_k \cdot 0.\underbrace{0...0}_{m-1} d_m$ is the standard product, and $k = m = 0$ means that $0.\underbrace{0...0}_{k-1} c_k = c_0$ and $0.\underbrace{0...0}_{m-1} d_m = d_0$. If either $c < 0$ or $d < 0$, then we compute $|c| \times_n |d|$ and define $c \times_n d = \pm |c| \times_n |d|$, where the sign $\pm$ is defined as usual. Note, if the content of at least one parentheses (in previous formula) is not in $W_n$, then $c \times_n d$ is not defined.

# Arithmetic - Division

- Division is defined to be

$$c \div_n d = \begin{cases} r, \text{ if } \exists! r \in W_n, r \times_n d = c \\ \text{not defined, if no such } r \text{ exists or not !} \end{cases}$$

## *Arithmetic - General*

- The arithmetic coincides with standard if the numbers are away from $W_n$ borders.
- If the borders are *touched*, then other properties arise.
- Mathematics based on idea of observers, given these arithmetic rules:
- Observer's Mathematics $-$ Mathematics of Relativity.
- For more info, visit `www.mathrelativity.com`.

# *Mathematics of Relativity*

- Various applications to Algebra, Geometry, Topology, and Analysis.
- In particular, a result in Logic: Axiom of Choice is invalid, e.g.
- $W_2$ contains 19,999 elements from the point of view of $W_n$-observer with large enough $n$.
- Then $W_2$-observer cannot choose *any* element from $W_2$ as any algorithm in $W_2$-world has no more than 99 steps.
- The (negative) solution to classical Fermat's problem requires Axiom of Choice to be valid.

# *Fermat's Problem Analogy*

Theorem: (Simple Case)

- For any $n$, $W_n$, $n \geq 2$;
- For any $m \in W_n \cap \mathbb{Z}$ with $m > 2$.
- There exists positive $a, b, c \in W_n$, such that
- $a^m +_n b^m = c^m$.

# Theorem (Simple Case) Proof

- Take $a = b = c = 0.\overbrace{0 \cdots 0}^{k}1$, $1 \leq k \leq n$, $k \times_n m > n$ ($km \in W_n$).
- Then $a^m = b^m = c^m = 0$, hence, $a^m +_n b^m = c^m$.
- Note: $a^m = \underbrace{(\cdots((a \times_n a) \times_n a) \times_n \cdots) \times_n a)}_{m}$
- Note: Power is not an associative operation.

# *Power Non-Associativity*

- For illustrative purposes, here is a $W_2$-example:
- $1.49 \times_2 1.49 = 2.14$.
- $1.49 \times_2 2.14 = 3.16$.
- $1.49 \times_2 3.16 = 4.67$, i.e.
- $((1.49 \times_2 1.49) \times_2 1.49) \times_2 1.49 = 4.67$, while
- $(1.49 \times_2 1.49) \times_2 (1.49 \times_2 1.49) = 4.57 \neq 4.67$.

# General Case Examples - $W_2$

- $1^3 +_2 1^3 = 1.28^3$.
- $1^{20} +_2 1^{20} = 1.05^{20}$.
- $1^{25} +_2 1^{25} = 1.04^{25}$.
- $1^{50} +_2 1^{50} = 1.02^{50}$.
- $1^3 +_2 1.21^3 = 1.41^3$.
- $1.2^3 +_2 1.03^3 = 1.41^3$.

## General Case Examples - $W_3$

- $1^{17} +_3 1^{17} = 1.044^{17}$.
- $1^{22} +_3 1^{22} = 1.034^{22}$.
- $1^{50} +_3 1^{50} = 1.016^{50}$.
- $1^{200} +_3 1^{200} = 1.005^{200}$.
- $1^{250} +_3 1^{250} = 1.004^{250}$.
- $1^{500} +_3 1^{500} = 1.002^{250}$.

## ***General Case Examples -*** $W_4$

- $1^{2000} +_4 1^{2000} = 1.0005^{2000}$.
- $1^{2500} +_4 1^{2500} = 1.0004^{2500}$.
- $1^{5000} +_4 1^{5000} = 1.0002^{5000}$.

# General Case Examples - $W_8$

- $1.85643209^5 +_8 1.55566643^5 = 1.98939654^5$.
- $1.00056781^4 +_8 1.42300976^4 = 1.50297066^4$.
- $1.85643209^4 +_8 1.67843218^4 = 2.10979538^4$.
- $1.8601023^3 +_8 1.35432561^3 = 2.07390372^3$.
- $1.02345678^3 +_8 1.25160402^3 = 1.44746886^3$.
- $1.13687002^3 +_8 1.57041392^3 = 1.74814264^3$.

# General Case Examples - $W_{16}$

- $1.4230990164830891^3 +_{16} 1.5704139255639073^3 = 1.8903509118894252^3$.