10. MAXWELL ELECTRODYNAMIC EQUATIONS - OBSERVER'S MATHEMATICS POINT OF VIEW

10.1 Tensors in electromagnetic fields theory

Let's consider now Observer's Mathematics point of view on Maxwell equations and their derivation.

We follow by classical way and start with four-potential of a field.

The coordinates of an event (ct, x, y, z) can be considered as the components of a four-dimensional radius vector (or, for short, a four-radius vector) in a four-dimensional space. We shall denote its components by x^i where the index i takes on the values $0, 1, 2, 3$, and

$$x^0 = ct, x^1 = x, x^2 = y, x^3 = z$$

In general a set of four quantities A^0, A^1, A^2, A^3 which transform like the components of the radius four-vector x^i under transformations of the four-dimensional coordinate system is called a four-dimensional vector (four-vector) $\mathbf{A}^i$. Under Lorentz transformations,

$$A^0 = \frac{A'^0 + \frac{V}{c}A'^1}{\sqrt{1 - \frac{V^2}{c^2}}}, A^1 = \frac{A'^1 + \frac{V}{c}A'^0}{\sqrt{1 - \frac{V^2}{c^2}}}, A^2 = A'^2, A^3 = A'^3$$

The properties of a particle with respect to interaction with the electromagnetic field are determined by a single parameter - the charge e of the particle, which can be either positive or negative (or equal to zero). The properties of the field are characterized by a four-vector $\mathbf{A}_i$, the four-potential, whose components are functions of the coordinates and time. Note for four-vector

$$\mathbf{A}_i = (A_0, A_1, A_2, A_3)$$

we can define four-vector

$$\mathbf{A}^i = (A^0, A^1, A^2, A^3)$$

with conditions

$$A^0 = A_0, A^1 = -A_1, A^2 = -A_2, A^3 = -A_3$$

The three space coordinates of the four-vector $\mathbf{A}^i$ form a three-dimensional vector $\mathbf{A}$ call the vector potential of the field. The time component is called the scalar potential; we denote it by $A^0 = \phi$. Thus

(L1)

$$\mathbf{A}^i = (\phi, \mathbf{A})$$

.

The quantities A^i are called the contravariant, and the A_i the covariant components of the four-vector which is considered as a tensor. By definition a four-dimensional tensor (four-tensor) of the second rank is a set of sixteen quantities A^{ik}, which under coordinate transformations transform like the products of components of two four-vectors. It is similarly defined four-tensors of higher rank.

We introduce the speed of particle

(L2)

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

where $\mathbf{r}$ is radius-vector of particle and

$$v = |\mathbf{v}|$$

.

When we go to **(L2)**, we note that the probability of the length of vector $\mathbf{v}$ ($v = |\mathbf{v}|$) exists is less than 1 (**SP4**)

Using action function for a charge in an electromagnetic field we get Lagrangian for a charge in an electromagnetic field:

(L3)

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} + \frac{e}{c}(\mathbf{A},\mathbf{v}) - e\phi$$

This function differs from a Lagrangian for a free particle by the terms

$$\frac{e}{c}(\mathbf{A},\mathbf{v}) - e\phi$$

which describe the interaction of the charge with the field.

Now let's consider **(L3)**.

For Observer's Mathematics point of view we proved the following theorem:

THEOREM 10.1. *In special relativity,* $P\left(L = -(m \times_n (c \times_n c)) \times_n \sqrt{1 - \frac{v \times_n v}{c \times_n c}}\right) < 1$, *where* P *is the probability.*

Based on this theorem we have to write down

(L3')

$$L = -m \times_n (c \times_n c) \times_n \sqrt{1 -_n \frac{v \times_n v}{c \times_n c}} +_n \xi_{31} +_n \frac{e}{c} \times_n (\mathbf{A},\mathbf{v}) -_n e \times_n \phi$$

where ξ_{31} is a random variable depends on n and m.

We assume that all elements of this equality belong to W_n.

We note that:

- the probability of $\sqrt{1 -_n \frac{v \times_n v}{c \times_n c}}$ exists is less than 1;
- the probability of $\frac{v \times_n v}{c \times_n c}$ exists is less than 1;
- the probability of $\frac{e}{c}$ exists is less than 1.

So, the probability of equality

$$L = -m \times_n (c \times_n c) \times_n \sqrt{1 -_n \frac{v \times_n v}{c \times_n c}} +_n \frac{e}{c} \times_n (\mathbf{A}, \mathbf{v}) -_n e \times_n \phi$$

is less than 1.

The derivative

(L4)

$$\partial L / \partial \mathbf{v} = \mathbf{P}$$

is the generalized momentum of the particle; carrying out the differentiation, we find

(L5)

$$\mathbf{P} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{e}{c}\mathbf{A} = \mathbf{p} + \frac{e}{c}\mathbf{A}$$

Here we have denoted by $\mathbf{p}$ the ordinary momentum of the particle.

The equations of motion of a charge in a given electromagnetic field are obtained by varying the action, i.e. they are given by the Lagrange equations:

(L6)

$$\frac{d}{dt}(\partial L / \partial \mathbf{v}) = \partial L / \partial \mathbf{r}$$

Further, we write

(L7)

$$\partial L / \partial \mathbf{r} = \nabla L = \frac{e}{c}(\mathbf{grad}\mathbf{A}, \mathbf{v}) - e\mathbf{grad}\phi$$

With correction of a formula of vector analysis

(L8)

$$\mathbf{grad}(\mathbf{a}, \mathbf{b}) = (\mathbf{a}, \nabla)\mathbf{b} + (\mathbf{b}, \nabla)\mathbf{a} + \mathbf{b} \times \mathbf{rot}\mathbf{a} + \mathbf{a} \times \mathbf{rot}\mathbf{b}$$

by **(N7)** we get

(L8')

$$\mathbf{grad}(\mathbf{a}, \mathbf{b}) = (\mathbf{a}, \nabla)\mathbf{b} +_n (\mathbf{b}, \nabla)\mathbf{a} +_n \mathbf{b} \times \mathbf{rot}\mathbf{a} +_n \mathbf{a} \times \mathbf{rot}\mathbf{b} +_n \xi_{24}$$

We assume that all elements of this equality belong to W_n.

And remembering that differentiation with respect to $\mathbf{r}$ is carried out for constant $\mathbf{v}$, and we get

(L9')

$$\partial L / \partial \mathbf{r} = \frac{e}{c} \times_n (\mathbf{v}, \nabla) \times_n \mathbf{A} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{rot}\mathbf{A} -_n e \times_n \mathbf{grad}\phi +_n \xi_{35}$$

where ξ_{35} is a random vector depends on n and m.

We assume that all elements of this equality beliong to W_n.

We note that the probability of $\frac{e}{c}$ exists is less than 1.

So, the probability of equality

$$\partial L/\partial \mathbf{r} = \frac{e}{c} \times_n (\mathbf{v}, \nabla) \times_n \mathbf{A} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{rot}\mathbf{A} -_n e \times_n \mathbf{grad}\phi$$

is less than 1.

So the Lagrange equation has the form:

(L10)

$$\frac{d}{dt}(\mathbf{p} + \frac{e}{c}\mathbf{A}) = \frac{e}{c}(\mathbf{v}, \nabla)\mathbf{A} + \frac{e}{c}\mathbf{v} \times \mathbf{rot}\mathbf{A} - e\mathbf{grad}\phi$$

But the total differential $(\frac{d\mathbf{A}}{dt})dt$ consists of two parts: the change $\partial\mathbf{A}/\partial t$ of the vector potential with time at a fixed point in space, and the change due to motion from one point in space to another at distance $d\mathbf{r}$. This second part is equal to $(d\mathbf{r}, \nabla)\mathbf{A}$.

But the Lagrange equation has the form:

(L10')

$$\frac{d}{dt}(\mathbf{p} +_n \frac{e}{c} \times_n \mathbf{A}) = \frac{e}{c} \times_n (\mathbf{v}, \nabla)\mathbf{A} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{rot}\mathbf{A} -_n e \times_n \mathbf{grad}\phi +_n \xi_{36}$$

where ξ_{36} is a random vector depends on n and m.

We assume that all elements of this equality beliong to W_n.

We note that the probability of $\frac{e}{c}$ exists is less than 1.

So, the probability of equality

$$\frac{d}{dt}(\mathbf{p} +_n \frac{e}{c} \times_n \mathbf{A}) = \frac{e}{c} \times_n (\mathbf{v}, \nabla)\mathbf{A} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{rot}\mathbf{A} -_n e \times_n \mathbf{grad}\phi$$

is less than 1.

We have in classical Mathematics

(L11)

$$\frac{d\mathbf{A}}{dt} = \partial\mathbf{A}/\partial t + (\mathbf{v}, \nabla)\mathbf{A}$$

And substituting this in **(L10)** , we find

(L12)

$$\frac{d\mathbf{p}}{dt} = -\frac{e}{c}\partial\mathbf{A}/\partial t - e\mathbf{grad}\phi + \frac{e}{c}\mathbf{v} \times \mathbf{rot}\mathbf{A}$$

But we get

(L12')

$$\frac{d\mathbf{p}}{dt} = -\frac{e}{c} \times_n \partial\mathbf{A}/\partial t -_n e \times_n \mathbf{grad}\phi +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{rot}\mathbf{A} +_n \xi_{37}$$

where ξ_{37} is a random vector depends on n and m.

We assume that all elements of this equality beliong to W_n.

We note that the probability of $\frac{e}{c}$ exists is less than 1.

So, the probability of equality

$$\frac{d\mathbf{p}}{dt} = -\frac{e}{c} \times_n \partial\mathbf{A}/\partial t -_n e \times_n \mathbf{grad}\phi +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{rot}\mathbf{A}$$

is less than 1.

This is the equation of motion of a particle in an electromagnetic field from Observer's Mathematics point of view.

This is the equation of motion of a particle in an electromagnetic field. On the left side stands the derivative of the particle's momentum with respect to the time. Therefore the expression on the right side is the force exerted on the charge in an electromagnetic field. We see that this force consists of two parts. The first part (first and second terms on the right side) does not depend on the velocity of the particle. The second part (third term) depends on the velocity, being proportional to the velocity and perpendicular to it. The force of the first type, per unit charge, is called the electric field intensity; we denote it by $\mathbf{E}$. So by definition,

(L13)

$$\mathbf{E} = -\frac{1}{c}\partial\mathbf{A}/\partial t - \mathbf{grad}\phi$$

The factor of $\frac{\mathbf{v}}{c}$ in the force of the second type, per unit charge, is called the magnetic field intensity. We designate it by $\mathbf{H}$. So by definition,

(L14)

$$\mathbf{H} = \mathbf{rot}\mathbf{A}$$

We assume that all elements of equalities **(L13')** and **(L14')** belong to W_n.

We note that the probability of $\frac{1}{c}$ exists is less than 1.

The equation of motion of a charge in an electromagnetic field can now be written as

(L15)

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}\mathbf{v} \times \mathbf{H}$$

But the equation of motion of a charge in an electromagnetic field can now be written as

(L15')

$$\frac{d\mathbf{p}}{dt} = e \times_n \mathbf{E} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{H} +_n \xi_{37} +_n \delta^2$$

where δ^2 is a random vector with coordinates $(\delta_2, \delta_2, \delta_2)$.

We assume that all elements of this equality belong to W_n.

We note that the probability of $\frac{e}{c}$ exists is less than 1.

So, the probability of equality

$$\frac{d\mathbf{p}}{dt} = e \times_n \mathbf{E} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{H}$$

is less than 1.

And now we have

(L15")

$$\frac{d\mathbf{p}}{dt} = e \times_n \mathbf{E} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{H} +_n \xi_{38}$$

where $\xi_{38} = \xi_{37} +_n \delta^2$ is a random vector depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equality

$$\frac{d\mathbf{p}}{dt} = e \times_n \mathbf{E} +_n \frac{e}{c} \times_n \mathbf{v} \times \mathbf{H}$$

is less than 1.

From the expressions **(L13)** and **(L14)**

it is easy to obtain equations containing only $\mathbf{E}$ and $\mathbf{H}$. To do this we find $\mathbf{rotE}$:

By **(L13)**, **(L14)** we get

(L16)

$$\mathbf{rotE} = -\frac{1}{c} \times_n (\partial/\partial t)\mathbf{rotA} -_n \mathbf{rotgrad}\phi +_n \xi_{39}$$

where ξ_{39} is a random vector depends on n and m.

We assume that all elements of this equality belong to W_n.

We note that the probability of $\frac{1}{c}$ exists is less than 1.

So, the probability of equality

$$\mathbf{rotE} = -\frac{1}{c} \times_n (\partial/\partial t)\mathbf{rotA} -_n \mathbf{rotgrad}\phi$$

is less than 1.

But in classical Mathematics the rot of any gradient is zero. Consequently,

(M1)

$$\mathbf{rotE} = -\frac{1}{c}\partial\mathbf{H}/\partial t$$

Taking the divergence of both sides of the equation

$$\mathbf{H} = \mathbf{rotA}$$

, and recalling that in classical Mathematics $div\mathbf{rot} = 0$, we find

(M2)

$$div\mathbf{H} = 0$$

.

In Observer's Mathematics we get

(M1')

$$\mathbf{rotE} = -\frac{1}{c} \times_n \partial\mathbf{H}/\partial t +_n \xi_{39} +_n \xi_{28}$$

and

(M1")

$$\mathbf{rotE} = -\frac{1}{c} \times_n \partial\mathbf{H}/\partial t +_n \xi_{40}$$

where $\xi_{40} = \xi_{39} +_n \xi_{28}$ is a random vector depends on n and m.

We assume that all elements of this equality beliong to W_n.

We note that the probability of $\frac{1}{c}$ exists is less than 1.

So, the probability of equality

$$\mathbf{rotE} = -\frac{1}{c} \times_n \partial\mathbf{H}/\partial t$$

is less than 1.

We also find

(M2')

$$div\mathbf{H} = \xi_{30}$$

We assume that all elements of this equality belong to W_n.

So, the probability of equality

$$div\mathbf{H} = 0$$

is less than 1.

Two last equations - **(M1”)** and **(M2’)** - are the first pair of Maxwell’s equations in Observer’s Mathematics.

We now introduce the notion

(L17)

$$F^{ik} = \partial A_k/\partial x_i - \partial A_i/\partial x_k$$

The antisymmetric tensor F^{ik} is called the electromagnetic field tensor.

The meaning of the individual components of the tensor F^{ik} is easily seen by substituting the values

$$\mathbf{A}_i = (\phi, -\mathbf{A})$$

in the definition **(L17)**.

The result can be written as a matrix in which the index $i = 0, 1, 2, 3$ labels the rows, and the index k the columns:

(L18) $F^{ik} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & H_z & -H_y \\ -E_y & -H_z & 0 & H_x \\ -E_z & H_y & -H_x & 0 \end{bmatrix}$

and

(L19) $F_{ik} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & H_z & -H_y \\ E_y & -H_z & 0 & H_x \\ E_z & H_y & -H_x & 0 \end{bmatrix}$

Instead of treating charges as points, for mathematical convenience we frequently consider them to be distributed continuously in space. Then we can introduce the "charge density" ρ such that ρdV is the charge contained in the volume dV. The density ρ is in general a function of the coordinates and the time. The integral of ρ over a certain volume is the charge contained in that volume.

Multiplying the equality

$$de = \rho dV$$

on both sides with dx^i (dx^i is 4-vector):

(L20)

$$dedx^i = \rho dV dx^i = \rho dV dt \frac{dx^i}{dt}$$

On the left stands a four-vector (since de is a scalar and dx^i is a four-vector). This means that the right side must be a four-vector. But $dVdt$ is a scalar, and so $\rho\frac{dx^i}{dt}$ is a four-vector. This vector (we denote it by $\mathbf{j}^i$) is called the current four-vector:

(L21)

$$\mathbf{j}^i = \rho \frac{dx^i}{dt}$$

The space components of this vector form a vector in ordinary space,

(L22)

$$\mathbf{j} = \rho \mathbf{v}$$

where $\mathbf{v}$ is the velocity of the charge at the given point. The vector $\mathbf{j}$ is called the current density vector. The time component of the current four-vector is $c\rho$. Thus

(L23)

$$\mathbf{j}^i = (c\rho, \mathbf{j})$$

This vector is called the current four-vector, which is considered as a tensor.

In finding the field equations with the aid of the principle of least action we must assume the motion of the charges to be given and vary only the potentials (which serve as the "coordinates" of the system) ; on the other hand, to find the equations of motion we assumed the field to be given and varied the trajectory of the particle.

We can write the variation of action S as

(L24)

$$\delta S = -\frac{1}{c}\int (\frac{1}{c} j^i \delta A_i - \frac{1}{4\pi} F^{ik} \partial/\partial x^k \delta A_i) d\Omega$$

The second of these integrals we integrate by parts

(L25)

$$\delta S = -\frac{1}{c}\int (\frac{1}{c} j^i + \frac{1}{4\pi} \partial F^{ik}/\partial x^k) \delta A_i d\Omega - \frac{1}{4\pi c}\int F^{ik} \delta A_i dS_k$$

By **(N2)** and theorem ???? we get

(L25')

$$\delta S = -\frac{1}{c}\int (\frac{1}{c} \times_n j^i +_n \frac{1}{4 \times_n \pi} \partial F^{ik}/\partial x^k +_n \xi^i_{41}) \times_n \delta A_i \times_n d\Omega -_n \frac{1}{(4 \times_n \pi) \times_n c}\int F^{ik} \times_n \delta A_i \times_n dS_k$$

where ξ^i_{41} is a random variable depends on n and m for each i.

We assume that all elements of this equality belong to W_n.

Also number π here is a standard π, but with only n digits in the decimal part of the fraction:

$$\pi = 3.\underbrace{14\ldots}_{n}$$

So, the probability of equation

$$\delta S = -\frac{1}{c}\int (\frac{1}{c} \times_n j^i +_n \frac{1}{4 \times_n \pi}\partial F^{ik}/\partial x^k) \times_n \delta A_i \times_n d\Omega -_n \frac{1}{(4 \times_n \pi) \times_n c}\int F^{ik} \times_n \delta A_i \times_n dS_k$$

is less than 1.

In the second term we must insert the values at the limits of integration. The limits for the coordinates are at infinity, where the field is zero. At the limits of the time integration, that is, at the given initial and final time values, the variation of the potentials is zero, since in accord with the principle of least action the potentials are given at these times. Thus the second term in **(L25')** is zero, and we find

(L26)

$$\int (\frac{1}{c}j^i + \frac{1}{4\pi}\partial F^{ik}/\partial x^k)\delta A_i d\Omega = 0$$

Since according to the principle of least action, the variations δA_i are arbitrary, the coefficients of the δA_i must be set equal to zero:

(L27)

$$\partial F^{ik}/\partial x^k = -\frac{4\pi}{c}j^i$$

We get

(L26')

$$\int (\frac{1}{c} \times_n j^i +_n \frac{1}{4 \times_n \pi}\partial F^{ik}/\partial x^k +_n \xi^i_{41})\delta A_i d\Omega = 0$$

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$\int (\frac{1}{c} \times_n j^i +_n \frac{1}{4 \times_n \pi}\partial F^{ik}/\partial x^k)\delta A_i d\Omega = 0$$

is less than 1.

Since according to the principle of least action, the variations δA_i are arbitrary, the coefficients of the δA_i must be set equal to zero:

(L27)

$$\partial F^{ik}/\partial x^k = -\frac{4\pi}{c}j^i$$

In Observer's Mathematics that means

(L27')

$$\partial F^{ik}/\partial x^k = -\frac{4 \times_n \pi}{c}j^i -_n \xi^i_{41}$$

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$\partial F^{ik}/\partial x^k = -\frac{4 \times_n \pi}{c} j^i$$

is less than 1.

Let us express these four ($i = 0, 1, 2, 3$) equations in three-dimensional form. By **(N3)** for $i = 1$ we get

(L28)

$$\frac{1}{c}\partial F^{10}/\partial t + \partial F^{11}/\partial x + \partial F^{12}/\partial y + \partial F^{13}/\partial z = -\frac{4\pi}{c} j^1$$

By **(L28)** we get

(L29)

$$\frac{1}{c}\partial E_x/\partial t - \partial H_z/\partial y + \partial H_y/\partial z = -\frac{4\pi}{c} j_x$$

In Observer's Mathematics that means

(L28')

$$\frac{1}{c} \times_n \partial F^{10}/\partial t +_n \partial F^{11}/\partial x +_n \partial F^{12}/\partial y +_n \partial F^{13}/\partial z = -\frac{4 \times_n \pi}{c} j^1 -_n \xi^1_{42}$$

where ξ^1_{42} is a random variable depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$\frac{1}{c} \times_n \partial F^{10}/\partial t +_n \partial F^{11}/\partial x +_n \partial F^{12}/\partial y +_n \partial F^{13}/\partial z = -\frac{4 \times_n \pi}{c} j^1$$

is less than 1.

By **(L28')** we get

(L29')

$$\frac{1}{c} \times_n \partial E_x/\partial t -_n \partial H_z/\partial y +_n \partial H_y/\partial z = -\frac{4 \times_n \pi}{c} \times_n j_x -_n \xi^1_{42}$$

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$\frac{1}{c} \times_n \partial E_x/\partial t -_n \partial H_z/\partial y +_n \partial H_y/\partial z = -\frac{4 \times_n \pi}{c} \times_n j_x$$

is less than 1.

This together with the two succeeding equations ($i = 2, 3$) can be written as one vector equation

(M3)

$$\mathbf{rotH} = \frac{1}{c}\partial\mathbf{E}/\partial t + \frac{4\pi}{c}\mathbf{j}$$

Finally, the fourth equation ($i = 0$) gives

(M4)

$$div\mathbf{E} = 4\pi\rho$$

And in Observer's Mathematics that means

(M3')

$$\mathbf{rotH} = \frac{1}{c} \times_n \partial\mathbf{E}/\partial t +_n \frac{4 \times_n \pi}{c} \times_n \mathbf{j} +_n \xi_{42}$$

where $\xi_{42} = (\xi_{42}^1, \xi_{42}^2, \xi_{42}^3)$ is a random vector depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$\mathbf{rotH} = \frac{1}{c} \times_n \partial\mathbf{E}/\partial t +_n \frac{4 \times_n \pi}{c} \times_n \mathbf{j}$$

is less than 1.

And

(M4')

$$div\mathbf{E} = (4 \times_n \pi) \times_n \rho +_n \xi_{42}^0$$

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$div\mathbf{E} = (4 \times_n \pi) \times_n \rho$$

is less than 1.

Tho last equations - **(M3')** and **(M4')** - are the second pair of Maxwell equations in Observer's Mathematics. Together with the first pair of Maxwell equations they completely determine the electromagnetic field, and are the fundamental equations of the theory of such fields, i.e. of electrodynamics from observer's Mathematics point of view.

From the Maxwell equations we can obtain the continuity equation.

(L30)

$$div\mathbf{rotH} = \frac{1}{c}(\partial/\partial t)div\mathbf{E} + \frac{4\pi}{c}div\mathbf{j}$$

But $div\mathbf{rot}\mathbf{H} = 0$ and by **(M4)**

we arrive at so-called equation of continuity, expressing the conservation of charge:

(L31)

$$div\mathbf{j} + \partial\rho/\partial t = 0$$

From Observer's Mathematics point of view we can obtain

(L30')

$$div\mathbf{rot}\mathbf{H} = \frac{1}{c} \times_n (\partial/\partial t) div\mathbf{E} +_n \frac{4 \times_n \pi}{c} div\mathbf{j} +_n \xi_{43}$$

where ξ_{43} is a random variable depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$div\mathbf{rot}\mathbf{H} = \frac{1}{c} \times_n (\partial/\partial t) div\mathbf{E} +_n \frac{4 \times_n \pi}{c} div\mathbf{j}$$

is less than 1.

And we arrive at so-called equation of continuity, expressing the conservation of charge:

(L31')

$$div\mathbf{j} +_n \partial\rho/\partial t +_n \xi_{44} = 0$$

where ξ_{44} is a random variable depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equation

$$div\mathbf{j} +_n \partial\rho/\partial t = 0$$

is less than 1.