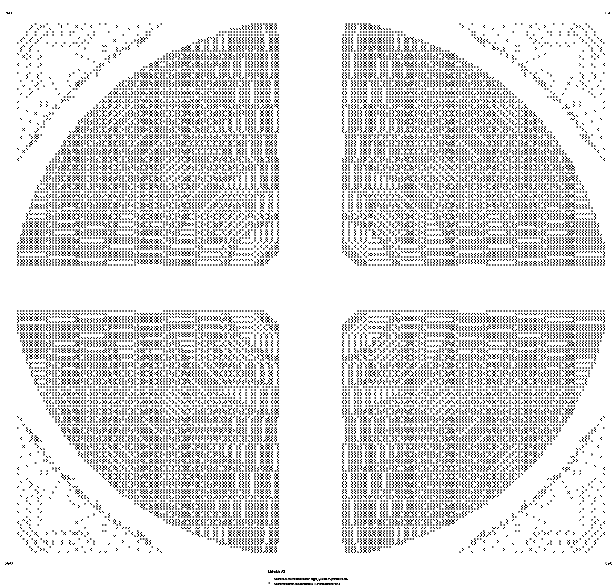
3. OBSERVER'S MATHEMATICS APPLICATIONS TO QUANTUM MECHANICS

3.1 Nadezhda effect

We consider an open square Q centered at the origin with sides of length 2 located on a plane $W_n \times W_n$. We will calculate the distance D between the origin $(0,0)$ and any point of Q as follows. $D = \rho((0,0),(x,y)) = \sqrt{x^2+y^2} = \sqrt{x \times_n x +_n y \times_n y}$, where $\sqrt{a} = b$ means $b \times_n b = a$, $x, y \in Q$, i.e., $|x| < 1$, $|y| < 1$.

The figure below contains an illustration of the fact that for some points on $W_n \times W_n$ the concept of distance from the origin does not exist; while for others it does exist. The illustration below is for $n = 3$ ($Q \subset W_3 \times W_3$). Points with no distance to the origin are indicated by black, while points where distance from the origin exists are indicated in white.



This means that the distance D does not always exist, i.e., not every segment on a plane has a length. This phenomenon occurs for all n. We call the presence of these "black holes" as the Nadezhda Effect. This effect gives us new possibilities for discovering physical processes and developing their mathematical models.

THEOREM 3.1. **Nadezhda Effect Theorem.** *For any positive integer n and W_n, consider the plane $W_n \times W_n = \{(x,y)\}, x, y \in W_n$ with standard Euclidean metric $d^2\left((x_1,y_1),(x_2,y_2)\right) = (x_1-x_2)^2 + (y_1-y_2)^2$. Next, consider any line $y = k \times_n x$, with $y, k, x \in W_n$. Then there is some point $(x_0, y_0) = (x_0, k \times_n x_0) \in W_n \times W_n$ such that $d((x_0,y_0),(0,0))$ does not exist.*

Lemma 1. If $y = 0.\underbrace{0...01}_{n} \times_n x$, then there exists $x_0 \in W_n$ such that ρ does not exist, where

$$\rho = \sqrt{x_0 \times_n x_0 + (0.\underbrace{0...01}_{n} \times_n x_0) \times_n (0.\underbrace{0...01}_{n} \times_n x_0)}$$

.

Lemma 2. If $y = k \times_n x$ with $0 \le k \le 1$, then there exists $x_0 \in W_n$ such that ρ does not exist, where

$$\rho = \sqrt{x_0 \times_n x_0 + (k \times_n x_0) \times_n (k \times_n x_0)}$$

.

Lemma 3. If $y = \underbrace{99...9}_{n}.\underbrace{99...9}_{n} \times_n x$, then there exists $x_0 \in W_n$ such that ρ does not exist, where

$$\rho = \sqrt{x_0 \times_n x_0 + (\underbrace{99...9}_{n}.\underbrace{99...9}_{n} \times_n x_0) \times_n (\underbrace{99...9}_{n}.\underbrace{99...9}_{n} \times_n x_0)}$$

.

Lemma 4. If $y = k \times_n x$ with $1 < k \le \underbrace{99...9}_{n}.\underbrace{99...9}_{n}$, then there exists $x_0 \in W_n$ such that ρ does not exist, where

$$\rho = \sqrt{x_0 \times_n x_0 + (k \times_n x_0) \times_n (k \times_n x_0)}$$

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3.2 Photoelectric effect from Observer's Mathematics point of view

In 1922, Albert Einstein received the Nobel Prize - not for his relativity theory, but for his interpretation of the photoelectric effect as being due to particle-like photons striking the surfaces of metals and ejecting electrons. But ironically it has been cogently argued that Einstein's conclusions were not fully justified. The theory of Lamb and Scully treated atoms quantum-mechanically, but regarded light as being a purely classical electromagnetic wave with no particle properties. Their conclusion was that the photoelectric effect does not constitute proof of the existence of photons.

Experimenters, therefore, led to design an experiment that asks whether light can be in two different places at the same time. The method is to place two detectors at widely separated locations, illuminate them both with the same light source, and ask whether they click at the same instant. Within the particle picture of light, they should not. The experimental apparatus required for such an experiment has to include: a light source, a half-silvered mirror and two detectors. Light falls on the half-silvered mirror, which acts as a beam splitter. If the incident light intensity is I, then behind the mirror the detectors each register an intensity $\frac{I}{2}$. Each detector responds with "click". Experimenters correlate these clicks by connecting them to a coincidence counter, which records a count only if both detectors click at the same moment. The results of such an experiment are conveniently analyzed in terms of the so-called anticorrelation parameter A:

$$A = \frac{P_c}{P_1 P_2}$$

where P_1 is the experimentally measured probability of detector 1 responding, P_2 is the experimentally measured probability of detector 2 responding, and P_c the probability of coincidence.

The quantity A has several properties that make it a particularly useful diagnostic in this situation. On the one hand, if light is composed of photons, the two detectors should never respond together, making P_c zero, so that A should be zero. If, on the other hand, light has no particle-like properties, the detectors should be perfectly capable of clicking together, and A can be non-zero. Indeed, if the detectors turn out to click randomly and independently of one another, experimenters can easily show that A will equal 1. Finally, a measured value of A greater than 1 would show the two detectors to be clicking together more often than purely random behavior would allow a "clustering" tendency of the clicks.

The Hanbury-Brown and Twiss experiment was done using this idea. And they used for anticorrelation parameter A calculation of the following formula:

$$A = \frac{\ll I^2 \gg}{\ll I \gg^2}$$

where $\ll I \gg$ is the average intensity over many instantaneous measurements, and $\ll I^2 \gg$ is the average of the intensity squared. The result shows that the expected anticorrelation parameter within the semi-classical theory (Lamb and Scully, Hanbury-Brown and Twiss) is simply the average of I squared as compared to the square of the average of I. And it was very easy to show that always $A > 1$. To see how that was done, begin with the simple case of a beam whose intensity fluctuates between only two values, I_1 and I_2. Defining x to be the ratio $\frac{I_2}{I_1}$, the averages are

$$\ll I^2 \gg = \frac{1}{2}(I_1^2 + I_2^2) = \frac{1}{2}(I_1^2)(1 + x^2)$$

and

$$\ll I \gg^2 = \left(\frac{1}{2}(I_1 + I_2)\right)^2 = I_1^2 \left(\frac{(1+x)}{2}\right)^2$$

But

$$\frac{1+x^2}{2} \geq \left(\frac{1+x}{2}\right)^2$$

because

$$2(1+x^2) \geq (1+x)^2$$

and

$$(1-x)^2 \geq 0$$

This result can be extended to a beam whose intensity fluctuates between any number of values by using Cauchy-Schwartz inequality:

$$\ll I^2 \gg \geq \ll I \gg^2$$

We now have the following

Theorem 3.2. *There are some values of light intensity where anticorrelation parameter* $A \in [0, 1)$.

Proof. For proof it is enough to find a corresponding example. Let's take $n = 2$, $I_1 = 0.2$, and $I_2 = 0.1$. We then have the following:

$$A \times_2 ((0.5 \times_2 (0.2 +_2 0.1)) \times_2 (0.5 \times_2 (0.2 +_2 0.1))) = 0.5 \times_2 ((0.2 \times_2 0.2) +_2 (0.1 \times_2 0.1))$$

which leads to

$$A \times_2 (0.15 \times_2 0.15) = 0.5 \times_2 (0.04 +_2 0.01)$$

which leads to

$$A \times_2 0.01 = 0$$

i.e. $A \in [0, 1)$. □

THEOREM 3.3. *There are some values of light intensity where anticorrelation parameter $A = 1$.*

Proof. For proof it is enough to find corresponding example. Let's take $n = 2$, $I_1 = 1.01$, and $I_2 = 1.02$. We then have the following:

$$A \times_2 ((0.5 \times_2 (1.01 +_2 1.02)) \times_2 (0.5 \times_2 (1.01 +_2 1.02)) = 0.5 \times_2 ((1.01 \times_2 1.01) +_2 (1.02 \times_2 1.02))$$

which leads to

$$A \times_2 (1 \times_2 1) = 0.5 \times_2 (1.02 +_2 1.04)$$

which leads to

$$A \times_2 1 = 1$$

i.e. $A = 1$. □

THEOREM 3.4. *There are some values of light intensity where anticorrelation parameter $A > 1$.*

Proof. For proof it is enough to find corresponding example. Let's take $n = 2$, $I_1 = 1.11$, $I_2 = 1.08$. We then have the following:

$$A \times_2 ((0.5 \times_2 (1.11 +_2 1.08)) \times_2 (0.5 \times_2 (1.11 +_2 1.08)) = 0.5 \times_2 ((1.11 \times_2 1.11) +_2 (1.08 \times_2 1.08))$$

which leads to

$$A \times_2 (1.05 \times_2 1.05) = 0.5 \times_2 (1.23 +_2 1.16)$$

which leads to

$$A \times_2 1.1 = 1.15$$

i.e. $A = 1.05$. □

These theorems show that with enough small intensities Einstein interpretation of the photoelectric effect as being due to particle-like photons striking the surfaces of metals and ejecting electrons is correct.

3.3 Dirac Equation for Free Electron

Let's consider Dirac equations for free electron in classical mathematics.

$$-m_0 c\psi_2 = \hbar\left(\frac{\partial\psi_{\hat{1}}}{\partial x^3} + \frac{\partial\psi_{\hat{1}}}{\partial x^0} + \frac{\partial\psi_{\hat{2}}}{\partial x^1} + i\frac{\partial\psi_{\hat{2}}}{\partial x^2}\right)$$

$$m_0 c\psi_1 = \hbar\left(\frac{\partial\psi_{\hat{1}}}{\partial x^1} - i\frac{\partial\psi_{\hat{1}}}{\partial x^2} - \frac{\partial\psi_{\hat{2}}}{\partial x^3} + \frac{\partial\psi_{\hat{2}}}{\partial x^0}\right)$$

$$-m_0 c\psi_{\hat{2}} = \hbar\left(\frac{\partial\psi_1}{\partial x^3} + \frac{\partial\psi_1}{\partial x^0} + \frac{\partial\psi_2}{\partial x^1} - i\frac{\partial\psi_2}{\partial x^2}\right)$$

$$m_0 c\psi_{\hat{1}} = \hbar\left(\frac{\partial\psi_1}{\partial x^1} + i\frac{\partial\psi_1}{\partial x^2} - \frac{\partial\psi_2}{\partial x^3} + \frac{\partial\psi_2}{\partial x^0}\right)$$

where $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, $\hbar = \frac{h}{2\pi}$, h is the Planck Constant, and $\hbar = 1.054\ldots \times 10^{-34}$ m^{2}kg/s, c is the speed of light, and $\psi_1, \psi_2, \psi_{\hat{1}}, \psi_{\hat{2}}$ are the spinors.

Now, consider the same equations in Observer's Mathematics, see **?**. $\Psi_1, \Psi_2 \in W_n$. Put $\psi_1 = \psi_1^a + i\psi_1^b$, $\psi_2 = \psi_2^a + i\psi_2^b$, $\psi_{\hat{1}} = \psi_{\hat{1}}^a + i\psi_{\hat{1}}^b$, and $\psi_{\hat{2}} = \psi_{\hat{2}}^a + i\psi_{\hat{2}}^b$.

After that, we have the following eight equations:

1. $-(m_0 \times_n c) \times_n \psi_2^a = \hbar \times_n \left(\left(\left(\frac{\partial\psi_{\hat{1}}^a}{\partial x^3} +_n \frac{\partial\psi_{\hat{1}}^a}{\partial x^0}\right) +_n \frac{\partial\psi_{\hat{2}}^a}{\partial x^1}\right) -_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^2}\right)$
2. $-(m_0 \times_n c) \times_n \psi_2^b = \hbar \times_n \left(\left(\left(\frac{\partial\psi_{\hat{1}}^b}{\partial x^3} +_n \frac{\partial\psi_{\hat{1}}^b}{\partial x^0}\right) +_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^1}\right) +_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^3}\right)$
3. $(m_0 \times_n c) \times_n \psi_1^a = \hbar \times_n \left(\left(\left(\frac{\partial\psi_{\hat{1}}^a}{\partial x^1} +_n \frac{\partial\psi_{\hat{1}}^b}{\partial x^2}\right) -_n \frac{\partial\psi_{\hat{2}}^a}{\partial x^3}\right) +_n \frac{\partial\psi_{\hat{2}}^a}{\partial x^0}\right)$
4. $(m_0 \times_n c) \times_n \psi_1^b = \hbar \times_n \left(\left(\left(\frac{\partial\psi_{\hat{1}}^b}{\partial x^1} -_n \frac{\partial\psi_{\hat{1}}^a}{\partial x^2}\right) -_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^3}\right) +_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^0}\right)$
5. $-(m_0 \times_n c) \times_n \psi_{\hat{2}}^a = \hbar \times_n \left(\left(\left(\frac{\partial\psi_1^a}{\partial x^3} +_n \frac{\partial\psi_1^a}{\partial x^0}\right) +_n \frac{\partial\psi_2^a}{\partial x^1}\right) +_n \frac{\partial\psi_2^b}{\partial x^2}\right)$
6. $-(m_0 \times_n c) \times_n \psi_{\hat{2}}^b = \hbar \times_n \left(\left(\left(\frac{\partial\psi_1^b}{\partial x^3} +_n \frac{\partial\psi_1^b}{\partial x^0}\right) +_n \frac{\partial\psi_2^b}{\partial x^1}\right) -_n \frac{\partial\psi_2^b}{\partial x^2}\right)$
7. $(m_0 \times_n c) \times_n \psi_{\hat{1}}^a = \hbar \times_n \left(\left(\left(\frac{\partial\psi_1^a}{\partial x^1} -_n \frac{\partial\psi_1^b}{\partial x^2}\right) -_n \frac{\partial\psi_2^a}{\partial x^3}\right) +_n \frac{\partial\psi_2^a}{\partial x^0}\right)$
8. $(m_0 \times_n c) \times_n \psi_{\hat{1}}^b = \hbar \times_n \left(\left(\left(\frac{\partial\psi_1^b}{\partial x^1} +_n \frac{\partial\psi_1^a}{\partial x^2}\right) -_n \frac{\partial\psi_2^b}{\partial x^3}\right) +_n \frac{\partial\psi_2^b}{\partial x^0}\right)$

We now have the following theorems.

THEOREM 3.5. *If m_0 is small enough such that $m_0 \times_n c = 0$ and $n > 35$ then*

$$\left(\left(\frac{\partial\psi_{\hat{1}}^a}{\partial x^3} +_n \frac{\partial\psi_{\hat{1}}^a}{\partial x^0}\right) +_n \frac{\partial\psi_{\hat{2}}^a}{\partial x^1}\right) -_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^2} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi_{\hat{1}}^b}{\partial x^3} +_n \frac{\partial\psi_{\hat{1}}^b}{\partial x^0}\right) +_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^1}\right) +_n \frac{\partial\psi_{\hat{2}}^b}{\partial x^3} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^a_{\hat{1}}}{\partial x^1} +_n \frac{\partial\psi^b_{\hat{1}}}{\partial x^2}\right) -_n \frac{\partial\psi^a_{\hat{2}}}{\partial x^3}\right) +_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^b_{\hat{1}}}{\partial x^1} -_n \frac{\partial\psi^b_{\hat{1}}}{\partial x^2}\right) -_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^3}\right) +_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^a_{1}}{\partial x^3} +_n \frac{\partial\psi^a_{1}}{\partial x^0}\right) +_n \frac{\partial\psi^a_{2}}{\partial x^1}\right) +_n \frac{\partial\psi^b_{2}}{\partial x^2} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^b_{1}}{\partial x^3} +_n \frac{\partial\psi^b_{1}}{\partial x^0}\right) +_n \frac{\partial\psi^b_{2}}{\partial x^1}\right) -_n \frac{\partial\psi^b_{2}}{\partial x^2} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^a_{1}}{\partial x^1} -_n \frac{\partial\psi^b_{1}}{\partial x^2}\right) -_n \frac{\partial\psi^a_{2}}{\partial x^3}\right) +_n \frac{\partial\psi^a_{2}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^b_{1}}{\partial x^1} +_n \frac{\partial\psi^a_{1}}{\partial x^2}\right) -_n \frac{\partial\psi^b_{2}}{\partial x^3}\right) +_n \frac{\partial\psi^b_{2}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

where any $* \in \{0, 1, \ldots, 9\}$ *and is random.*

THEOREM 3.6. *Let* $n > 35$, $0 < k < n$, *and* $m_0 \times_n c = 0.\overbrace{\underbrace{0\ldots0}_{k}*\ldots*}^{n}$, *also let*

$$\left(\left(\frac{\partial\psi^a_{\hat{1}}}{\partial x^3} +_n \frac{\partial\psi^a_{\hat{1}}}{\partial x^0}\right) +_n \frac{\partial\psi^a_{\hat{2}}}{\partial x^1}\right) -_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^2} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^b_{\hat{1}}}{\partial x^3} +_n \frac{\partial\psi^b_{\hat{1}}}{\partial x^0}\right) +_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^1}\right) +_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^3} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^a_{\hat{1}}}{\partial x^1} +_n \frac{\partial\psi^b_{\hat{1}}}{\partial x^2}\right) -_n \frac{\partial\psi^a_{\hat{2}}}{\partial x^3}\right) +_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^b_{\hat{1}}}{\partial x^1} -_n \frac{\partial\psi^b_{\hat{1}}}{\partial x^2}\right) -_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^3}\right) +_n \frac{\partial\psi^b_{\hat{2}}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^a_{1}}{\partial x^3} +_n \frac{\partial\psi^a_{1}}{\partial x^0}\right) +_n \frac{\partial\psi^a_{2}}{\partial x^1}\right) +_n \frac{\partial\psi^b_{2}}{\partial x^2} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^b_{1}}{\partial x^3} +_n \frac{\partial\psi^b_{1}}{\partial x^0}\right) +_n \frac{\partial\psi^b_{2}}{\partial x^1}\right) -_n \frac{\partial\psi^b_{2}}{\partial x^2} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi^a_{1}}{\partial x^1} -_n \frac{\partial\psi^b_{1}}{\partial x^2}\right) -_n \frac{\partial\psi^a_{2}}{\partial x^3}\right) +_n \frac{\partial\psi^a_{2}}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

$$\left(\left(\frac{\partial\psi_1^b}{\partial x^1} +_n \frac{\partial\psi_1^a}{\partial x^2}\right) -_n \frac{\partial\psi_2^b}{\partial x^3}\right) +_n \frac{\partial\psi_2^b}{\partial x^0} = 0.\overbrace{\underbrace{0\ldots0}_{n-35}*\ldots*}^{n}$$

where any $* \in \{0, 1, \ldots, 9\}$, *then*

$$\psi_1^a = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_1^b = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_2^a = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_2^b = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_{\hat{1}}^a = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_{\hat{1}}^b = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_{\hat{2}}^a = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n},$$

$$\psi_{\hat{2}}^b = 0.\overbrace{\underbrace{0\ldots0}_{n-k}*\ldots*}^{n}$$

where any $* \in \{0, 1, \ldots, 9\}$ *and is random. Thus,* ψ_1^a, ψ_1^b,ψ_2^a, ψ_2^b, $\psi_{\hat{1}}^a$, $\psi_{\hat{1}}^b$, $\psi_{\hat{2}}^a$, *and* $\psi_{\hat{2}}^b$ *are random variables.*

3.4 Solitary waves and dispersive equations from Observerỹ Mathematics point of view

In classical physics, it has been realized for centuries that the behavior of idealized vibrating media (such as waves on string, on a water surface, or in air), in the absence of friction or other dissipative forces, can be modeled by a number of partial differential equations, known collectively as dispersive equations. Model examples of such equations include the following:

- The free wave equation

$$u_{tt} - c^2\Delta u = 0$$

where $u : R \times R^d \to R$ represents the amplitude $u(t, x)$ of a wave at a point in spacetime with d spatial dimensions, $\Delta = \sum_{j=1}^{d} \frac{\delta^2}{\delta x_j^2}$ is the spatial Laplacian on R^d, u_{tt} is short for $\frac{\delta^2 u}{\delta t^2}$, and $c > 0$ is a fixed constant.

- The linear Schrodinger equation

$$i\hbar u_t + \frac{\hbar^2}{2m} \Delta u = V u$$

where $u : R \times R^d \to R$ is the wave function of a quantum particle, $\hbar, m > 0$ are physical constants and $V : R^d \to R$ is a potential function, which we assume to depend only on the spatial variable x.

- The Airy equation

$$u_t + u_{xxx} = 0$$

where $u : R \times R \to R$ is a scalar function.

- The Korteweg-de Vries equation

$$u_t + u_{xxx} + 6uu_x = 0$$

which is a more refined version of the Airy equation in which the first nonlinear term is retained.

The theory of linear dispersive equations predicts that waves should spread out and disperse over time. However, it is a remarkable phenomenon, observed both in theory and practice, that once nonlinear effects are taken into account, solitary wave and soliton solutions can be created, which can be stable enough to persist indefinitely. In this section we consider some properties of these equations from Observer's Mathematics point of view.

3.4.1 Free Wave Equation

We consider the case when $d = 1$, i.e., $u : W_n \times W_n \to W_n$, from W_m-observer point of view, with $m > n$, where $W_n \times W_n$ means Cartesian product of W_n with itself. The free wave equation may be written as

$$u_{tt} -_n ((c \times_n c) \times_n u_{xx}) = 0$$

Then we have the following

Theorem 3.7. *Let*

$$c = c_0.c_1 \ldots c_k c_{k+1} \ldots c_n$$

and

$$u_{xx} = \pm u_0^{xx}.u_1^{xx} \ldots u_l^{xx} u_{l+1}^{xx} \ldots u_n^{xx}$$

with $2k < n$, $l < n$, $c_0 = c_1 = \ldots = c_k = 0$, $c_{k+1} \neq 0$, $u_0^{xx} = u_1^{xx} = \ldots = u_l^{xx} = 0$ *and* $u < k + l + 2$, *then* $u_{tt} = 0$.

Next, we have the following

Theorem 3.8. *If* $d_0 \geq \underbrace{9 \ldots 9}_{p}$, *with* $0 < p \leq n$ *and* $u_0^{xx} \geq \underbrace{9 \ldots 9}_{q}$, *with* $0 < q \leq n$ *and* $n < p + q$, *then there is no* u_{tt}, *such that* $u_{tt} = ((c \times_n c) \times_n u_{xx})$.

3.4.2 Schrodinger Equation

Consider the following: $-(\hbar \times_n \hbar) \times_n \Psi_{xx} +_n ((2 \times_n m) \times_n V) \times_n \Psi = i((2 \times_n m) \times_n \hbar)\Psi_t$, where $\Psi = \Psi(x,t)$, $\hbar$ is the Planck's Constant, $\hbar = 1.054571628(53) \times 10^{-34}\ m^2kg/s$. $\Psi \in CW_n$, $\Psi = \Psi^a + i\Psi^b$. In the following statements we speak about $\Psi^a_x x$, $\Psi^b_x x$, Ψ^a_t, Ψ^b_t, Ψ^a, and Ψ^b.

Then we have the following

Theorem 3.9. *Let $36 < n < 68$, $m = m_0.m_1 \ldots m_k m_{k+1} \ldots m_n$, with $m \in W_n$, $m_0 = m_1 = \ldots = m_k = 0$, $m_{k+1} \neq 0$, $k + 35 < n$, $V = 0$, then $\Psi_t = \Psi^0_t.\Psi^1_t \ldots \Psi^l_t \Psi^{l+1}_t \ldots \Psi^n_t$ and $\Psi^0_t = \ldots \Psi^l_t = 0$, $\Psi^{l+1}_t, \ldots, \Psi^n_t$ are free and in $\{0, 1, \ldots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with $\Psi_t \in \{(0.\overbrace{\underbrace{0\ldots0}_{l} * \ldots *}^{n})\}$, where $* \in \{0, 1, \ldots, 9\}$.*

Corollary 3.10. *Let $36 < n < 68$, $m = m_0.m_1 \ldots m_k m_{k+1} \ldots m_n$, with $m \in W_n$, $m_0 = m_1 = \ldots = m_k = 0$, $m_{k+1} \neq 0$. Also, let $V = \upsilon_0.\upsilon_1 \ldots \upsilon_s \upsilon_{s+1} \ldots \upsilon_n$, with $V \in W_n$, $\upsilon_0 = \upsilon_1 = \ldots = \upsilon_s = 0$, $\upsilon_{s+1} \neq 0$, with $\left\{ \begin{array}{ccc} k+35 & < & n \\ k+s+2 & > & n \end{array} \right.$, then $\Psi_t = \Psi^0_t.\Psi^1_t \ldots \Psi^l_t \Psi^{l+1}_t \ldots \Psi^n_t$ and $\Psi^0_t = \ldots \Psi^l_t = 0$, $\Psi^{l+1}_t, \ldots, \Psi^n_t$ are free and in $\{0, 1, \ldots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with $\Psi_t \in \{(0.\overbrace{\underbrace{0\ldots0}_{l} * \ldots *}^{n})\}$, where $* \in \{0, 1, \ldots, 9\}$.*

3.4.3 Two-Slit Interference

Quantum mechanics treats the motion of an electron, neutron or atom by writing down the Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{\delta^2\Psi}{\delta x^2} + V\Psi = i\hbar\frac{\delta\Psi}{\delta t}$$

where m is the particle mass and V is the external potential acting on the particle. As these particles pass through the two slits of any of the experiments they are moving freely; we, therefore, set $V = 0$ in the Schrodinger equation.

Now, consider the following:

$$-(\hbar \times_n \hbar) \times_n \Psi_{xx} +_n ((2 \times_n m) \times_n V) \times_n \Psi = i((2 \times_n m) \times_n \hbar)\Psi_t$$

where $\Psi = \Psi(x,t)$, $\hbar$ is the Planck's Constant, $\hbar = 1.054571628(53) \times 10^{-34}\ m^2kg/s$. Then we have the following

Theorem 3.11. *Let $36 < n < 68$, $m = m_0.m_1 \ldots m_k m_{k+1} \ldots m_n$, with $m \in W_n$, $m_0 = m_1 = \ldots = m_k = 0$, $m_{k+1} \neq 0$, $k + 35 < n$, $V = 0$, then $\Psi_t = \Psi^0_t.\Psi^1_t \ldots \Psi^l_t \Psi^{l+1}_t \ldots \Psi^n_t$ and $\Psi^0_t = \ldots \Psi^l_t = 0$, $\Psi^{l+1}_t, \ldots, \Psi^n_t$ are free and in $\{0, 1, \ldots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with $\Psi_t \in \{(0.\overbrace{\underbrace{0\ldots0}_{l} * \ldots *}^{n})\}$, where $* \in \{0, 1, \ldots, 9\}$.*

The wave at the point of combination will be the sum of those from each slit. If Ψ_1 is the wave from slit 1 and Ψ_2 is the wave from slit 2, then $\Psi = \Psi_1 + \Psi_2$. The result gives the predicted interference pattern. Then we have

$$\Psi_{1t} = \Psi_{1t}^0 . \Psi_{1t}^1 \ldots \Psi_{1t}^l \Psi_{1t}^{l+1} \ldots \Psi_{1t}^n$$

$$\Psi_{2t} = \Psi_{2t}^0 . \Psi_{2t}^1 \ldots \Psi_{2t}^l \Psi_{2t}^{l+1} \ldots \Psi_{2t}^n$$

$$\Psi_{1t}^0 = \ldots = \Psi_{1t}^l = 0$$

Where $\Psi_{1t}^{l_1+1}, \ldots, \Psi_{1t}^n$ are free and in $\{0, 1, \ldots, 9\}$. and

$$\Psi_{2t}^0 = \ldots = \Psi_{2t}^l = 0$$

Where $\Psi_{2t}^{l_2+1}, \ldots, \Psi_{2t}^n$ are free and in $\{0, 1, \ldots, 9\}$ where $l = n - k - 36$.

Now we have the following

THEOREM 3.12.

1. *If* $\Psi_{1t}^{l+1} + \Psi_{2t}^{l+1} > 9$, *then* $\Psi_1 + \Psi_2$ *is not a wave.*
2. *If* $\Psi_{1t}^{l+1} + \Psi_{2t}^{l+1} < 9$, *then* $\Psi_1 + \Psi_2$ *is a wave.*
3. *If* $\Psi_{1t}^{l+1} + \Psi_{2t}^{l+1} = 9$, *then* $\Psi_1 + \Psi_2$ *may or may not be a wave.*

3.4.4 Airy and Korteweg-de Vries Equations

If $u : W_n \times W_n \to W_n$ then the Airy equation may be written as

$$u_t +_n u_{xxx} = 0$$

and Korteweg-de Vries equation may be written as

$$(u_t +_n u_{xxx}) +_n 6(u \times_n u_x) = 0$$

Then we have the following

THEOREM 3.13. *Let*

$$u = u_0 . u_1 \ldots u_k u_{k+1} \ldots u_n$$

and

$$u_x = u_0^x . u_1^x \ldots u_l^x u_{l+1}^x \ldots u_n^x$$

with $k < n$, $l \leq n$ *and* $u_0 = u_1 = \ldots = u_k = 0$ *and* $u_0^x = u_1^x = \ldots = u_l^x = 0$ *and* $k + l > n$, *then Airy equation and Korteweg-de Vries equation have the solution.*

3.4.5 Schwartzian Derivative

The Schwartzian derivative $S(f(x))$ is defined as

$$S(f(x)) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2$$

Here $f(x)$ is a function in one real variable and $f'(x), f''(x), f'''(x)$ are its derivatives. The Schwartzian derivative is ubiquitous and tends to appear in seemingly unrelated fields of Mathematics including classical complex analysis, differential equations, and one-dimensional analysis, as well as more recently, Teichmüller Theory, integrable systems, and conformal field theory. For example, let's consider the Lorentz plane with the metric $g = dxdy$ and a curve $y = f(x)$. If $f'(x) > 0$, then its Lorentz curvature can be easily computed via

$$\rho(x) = f''(x)(f'(x))^{-\frac{3}{2}}$$

and the Schwartzian enters the game when one computes $\rho' = \frac{S(f)}{\sqrt{f'}}$. Thus, informally speaking, the Schwartzian derivative is curvature.

Consider now the Schwartzian curvature from Observer's Mathematics point of view.

Now we have the following

THEOREM 3.14. *If $S(f(x))$ exists, then*

- *$S(f(x))$ is a random variable.*
- *$|S(f(x)| \leq 10^{l-k+1}$, where*

$$(2 \times_n (f'(x) \times_n f'(x))) = 0.\underbrace{0\ldots0a_l}_{l}a_{l+1}\ldots a_n$$

with $a_l \neq 0$ and

$$(2 \times_n (f'''(x) \times_n f'(x))) -_n (3 \times_n (f''(x) \times_n f''(x))) = \pm 0.\underbrace{0\ldots0b_k}_{k}b_{k+1}\ldots b_n$$

with $b_k \neq 0$ and $1 < l, k < n$.

3.4.6 Newton equation

Let $F(x,t) = m \times_n \ddot{x}$. Then we have the following

THEOREM 3.15. *If the body with mass $m = m_0.m_1\ldots m_k m_{k+1}\ldots m_n$, with $m \in W_n$, moves with acceleration $\ddot{x}$, $|\ddot{x}| = \ddot{x}_0.\ddot{x}_1\ldots\ddot{x}_l\ddot{x}_{l+1}\ldots\ddot{x}_n$, with $\ddot{x} \in W_n$, and $m_0 = m_1 = \ldots = m_k = 0$, $m_{k+1} \neq 0$, $k < n$, $\ddot{x}_0 = \ddot{x}_1 = \ldots = \ddot{x}_l = 0$, $l < n$, $k + l + 2 \in W_n$, $n < k + l + 2 \leq q$, then $F(x,t) = 0$.*

COROLLARY 3.16. *If $l = n - 1$ and $k = 0$, i.e., $m < 1$, then $F(x,t)$.*

THEOREM 3.17. *If $l = n - 1$ and $\ddot{x}_n \neq 0$ then $|F(x,t)| < 9$.*

THEOREM 3.18. *If $m_0 \geq \underbrace{9\ldots9}_{p}$, $0 < p \leq n$, $\ddot{x}_0 \geq \underbrace{9\ldots9}_{r}$, $0 < r \leq n$, $n < p + r \leq q$, then there is no force $F(x,t)$, such that $F(x,t) = m \times_n \ddot{x}$.*

3.4.7 Geodesic equation

Consider the following:

$$\ddot{x}^i +_n \sum_j {}^n\sum_k {}^n\Gamma^i_{jk} \times_n (\dot{x}^j \times_n \dot{x}^k) = 0$$

with $j, k \in G$. Then we have the following

THEOREM 3.19. *If $\dot{x}^p = \dot{x}^p_0.\dot{x}^p_1 \ldots \dot{x}^p_l \dot{x}^p_{l+1} \ldots \dot{x}^p_n$, with $p \in G$, $\dot{x}^p_0 = \dot{x}^p_1 = \ldots = \dot{x}^p_l = 0$, $0 \le l \le n$, $n < 2l \le q$, then we have $\ddot{x}^i = 0$, i.e., the geodesic curve is a line.*

3.4.8 Wave-Particle Duality for Single Photons

The connection between interference as a characteristic of waves and particles was noticed by de Broglie. He connected particle to wave mechanics. He proposed that particles behave as if they possessed a wavelength that was inversely proportional to their momentum, mV, and that the constant of proportionality was Planck's constant $\hbar$:

$$\lambda = \frac{\hbar}{mV}$$

I.e. we have

$$\lambda m V = \hbar$$

We have the following equation in Observer's Mathematics: $\lambda \times_n (m \times_n V) = \hbar$ with $\hbar = 1.\ldots \times 10^{-34}$ for $n > 60$. And now we have the following

THEOREM 3.20. *If $m \times_n V = c_0.c_1 \ldots c_k c_{k+1} \ldots c_n$ with $c_0 = c_1 = \ldots = c_k = 0$ and $c_{k+1} \neq 0$, with $k < n$ and $\lambda = \lambda_0.\lambda_1 \ldots \lambda_m \lambda_{m+1} \ldots \lambda_n$ with $m < n$ and $m + k > n$, then $\lambda_{m+1}, \ldots, \lambda_n$ are free and in $\{0, 1, \ldots, 9\}$ and $\lambda_0.\lambda_1 \ldots \lambda_m \times_n 0.\underbrace{0 \ldots 0}_{k} c_{k+1} \ldots c_n = \hbar$.*

3.4.9 Uncertainty Principle

Using the de Broglie relationship between momentum and wave number, $p = (\hbar)k$, we can obtain the position-momentum uncertainty relationship:

$$\Delta p \cdot \Delta x = \hbar$$

THEOREM 3.21. *Let $\Delta p = p_0.p_1 \ldots p_k p_{k+1} \ldots p_n$ with $k < n$ and $\Delta x = x_0.x_1 \ldots x_l x_{l+1} \ldots x_n$ with $l < n$. Then*

1. *If $p_0 = p_1 = \ldots = p_k = 0$ and $k + l > n$, then $x_{l+1}, \ldots, x_n$ are free and in $\{0, 1, \ldots, 9\}$.*
2. *If $x_0 = x_1 = \ldots = x_l = 0$ and $k + l > n$, then $p_{k+1}, \ldots, p_n$ are free and in $\{0, 1, \ldots, 9\}$.*
3. *If $p_0 = p_1 = \ldots = p_k = 0$ and $x_0 = x_1 = \ldots = x_l = 0$ then $k + l \le 34$.*