8. KEPLER'S FIRST LAW, NEWTON'S LAW OF GRAVITATION AND NEWTON'S SECOND LAW FROM OBSERVER'S MATHEMATICS POINT OF VIEW

The system under consideration contains a test particle (the planet) moving under the attraction of a massive body (the Sun). The planet's mass is assumed to be so small relative to that of the Sun that the Sun remains fixed in space. Also, the effects of all other planets are neglected. By Newton's second law

$$\mathbf{F} = m \times_n \mathbf{a}$$

where $\mathbf{F}$ is the force experienced by the planet, m is its mass, and $\mathbf{a}$ is its acceleration. And Newton's law of gravitation

$$\mathbf{F} \times_n r^2 = -(G \times_n (M \times_n m)) \times_n \mathbf{u}$$

$$\mathbf{F} \times_n r^3 = -(G \times_n (M \times_n m)) \times_n \mathbf{r}$$

$\mathbf{F}$ is the force experienced by the planet, G denotes the gravitational constant, $\mathbf{r} = \mathbf{r}(t)$ is the planet's position vector (origin is a Sun position), r is a length of vector $\mathbf{r}$, $\mathbf{u}$ is a unit vector in the direction of $\mathbf{r}$, and M and m are the masses of the Sun and planet, respectively. Further, $\mathbf{v} = \mathbf{r}'$, and $\mathbf{a} = \mathbf{r}''$, $\mathbf{r} = r \times_n \mathbf{u}$.

We note:

- probability of existing length r for vector $\mathbf{r}$ is less then 1 (**SP4**);

- probability of existing unit vector $\mathbf{u}$ on direction $\mathbf{r}$ is less then 1 (**SP4**);

- probability of existing inverse numbers $\frac{1}{r}$ is less then 1 (**I1**).

- probability of

$$-(G \times_n ((M \times_n m) \times_n (\frac{1}{r})^3) \times_n \mathbf{r} =$$

$$= -(G \times_n ((M \times_n m) \times_n (\frac{1}{r})^2) \times_n \mathbf{u}$$

is less then 1 (**LS5**).

- probability of

$$-(G \times_n ((M \times_n m) \times_n (\frac{1}{r})^3) \times_n \mathbf{r} =$$

$$= -(m \times_n ((G \times_n M) \times_n (\frac{1}{r})^3)) \times_n \mathbf{r}$$

is less then 1 (**LS5**).

First we have to understand does the planet move in a plane or not? Equating the $\mathbf{F}s$ gives us

(NK1)

$$(m \times_n \mathbf{a}) \times_n r^2 = -(m \times_n (G \times_n M))) \times_n \mathbf{u}$$

(NK1')

$$(m \times_n \mathbf{a}) \times_n r^3 = -(m \times_n (G \times_n M))) \times_n \mathbf{r}$$

a and **r** are parallel, so

(NK2)

$$\mathbf{r} \times \mathbf{a} = \mathbf{0}$$

We consider the equality **(NK2)** as a first approach.

We note:

- probability of this equality is less than 1 **(CP6)**. That means

(NK2')

$$\mathbf{r} \times \mathbf{a} = \boldsymbol{\Delta}_1$$

$\boldsymbol{\Delta}_1$ is a random vector depends on n $(\mathbf{r}, \mathbf{a}, \boldsymbol{\Delta}_1 \in E_3W_n)$

(NK2")

$$(\mathbf{r} \times \mathbf{v})' =$$

$$= \mathbf{r}' \times \mathbf{v} +_n \mathbf{r} \times \mathbf{v}' = \mathbf{v} \times \mathbf{v} +_n \mathbf{r} \times \mathbf{a} = \mathbf{0} +_n \mathbf{0} = \mathbf{0}$$

So, we can conclude that $\mathbf{r}x\mathbf{v}$ is a constant vector, say **h**:

(NK2"')

$$(\mathbf{r} \times \mathbf{v}) = \mathbf{h}$$

We consider the equalities **(NK2")** and **(NK2"')** as a first approach.

We note:

- probability of these equalities is less than 1 **(D5)**, **(CP6)**.

- probability of "$\mathbf{h} = \mathbf{const}$" is less than 1 **(D3)**. We can assume that **r** and **v** are not parallel. But probability of "$\mathbf{h} \neq \mathbf{0}$" is less than 1 **(CP6)**. That means

(NK3)

$$(\mathbf{r} \times \mathbf{v})' =$$

$$= (\mathbf{r}' \times \mathbf{v} +_n \mathbf{r} \times \mathbf{v}') +_n \Delta_2 =$$

$$= \mathbf{v} \times \mathbf{v} +_n \mathbf{r} \times \mathbf{a} +_n \Delta_2 = \mathbf{0} +_n \Delta_1 +_n \Delta_2 = \Delta_1 +_n \Delta_2$$

So,

(NK3')

$$\mathbf{r} \times \mathbf{v} = \mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3$$

where **h** is a constant vector, and we can assume that $\mathbf{h} \neq \mathbf{0}, t \in W_n$. $\Delta_1, \Delta_2, \Delta_3$ are random vectors depend on n.

(NK4)

$$(\mathbf{r}, \mathbf{h}) = 0$$

We consider the equality **(NK4)** as a first approach. Probability of "**r** and **h** are perpendicular" is less than 1 **(CP6)**. That means

(NK4')

$$(\mathbf{r},\mathbf{h}) = \Delta_4$$

, and

(NK4")

$$(\mathbf{r},\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) =$$
$$= (\mathbf{r},\mathbf{h}) +_n t \times_n (\mathbf{r},\Delta_1 +_n \Delta_2) +_n (\mathbf{r},\Delta_3) +_n \Delta_5 =$$
$$= t \times_n (\mathbf{r},\Delta_1 +_n \Delta_2) +_n (\mathbf{r},\Delta_3) +_n \Delta_4 +_n \Delta_5$$

Δ_4, Δ_5 are random variables depend on n.

Planetary motion, according to Newton's laws, is planar, because the planet always lies in a plane through the origin perpendicular to **h**. But from Observer's Mathematics point of view the planet orbit is a curve on surface generated by vector **r**, and this surface is not a plane. But deviation between these surface and plane is decreasing with growing n.

Now we can prove examine the shape of the orbit within this surface. We have

(NK5)

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} = \mathbf{r} \times \mathbf{r}' =$$
$$= (r \times_n \mathbf{u}) \times (r \times_n \mathbf{u})' = (r \times_n \mathbf{u}) \times (r \times_n \mathbf{u}' +_n r' \times_n \mathbf{u}) =$$
$$= r^2 \times_n (\mathbf{u} \times \mathbf{u}') +_n (r \times_n r') \times_n (\mathbf{u} \times \mathbf{u}) = r^2 \times_n (\mathbf{u} \times \mathbf{u}')$$

We consider the equality **(NK5)** as a first approach.

We note:

- $r^2 = r \times_n r$;

- probability of equality "$\mathbf{r} \times \mathbf{r}' = (r \times_n \mathbf{u}) \times (r \times_n \mathbf{u})'$" is less than 1 **(CP4)**;

- probability of equality "$(r \times_n \mathbf{u})' = r \times_n \mathbf{u}' +_n r' \times_n \mathbf{u}$" is less than 1 **(D2)**;

- probability of equality "

$$(r \times_n \mathbf{u}) \times (r \times_n \mathbf{u}' +_n r' \times_n \mathbf{u}) =$$
$$= r^2 \times_n (\mathbf{u} \times \mathbf{u}') +_n (r \times_n r') \times_n (\mathbf{u} \times \mathbf{u})$$

" is less than 1 **(CP3, CP4)**. That means

(NK5')

$$\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3 = \mathbf{r} \times \mathbf{v} = \mathbf{r} \times \mathbf{r}' =$$
$$= (r \times_n \mathbf{u}) \times (r \times_n \mathbf{u})' =$$
$$= (r \times_n \mathbf{u}) \times ((r \times_n \mathbf{u}' +_n r' \times_n \mathbf{u}) +_n \Delta_6) =$$
$$= ((r^2 \times_n (\mathbf{u} \times \mathbf{u}') +_n (r \times_n r') \times_n (\mathbf{u} \times \mathbf{u})) +_n r \times_n (\mathbf{u} \times \Delta_6) +_n \Delta_7 =$$

$$= (r^2 \times_n (\mathbf{u} \times \mathbf{u}') +_n r \times_n (\mathbf{u} \times \Delta_6)) +_n \Delta_7$$

Δ_6, Δ_7 are random vectors depend on n.

So,

(NK6)

$$\mathbf{a} \times \mathbf{h} = ((-G \times_n M) \times_n (\frac{1}{r})^2) \times_n \mathbf{u} \times r^2 \times_n (\mathbf{u} \times \mathbf{u}') =$$

$$= (-G \times_n M) \times_n \mathbf{u} \times (\mathbf{u} \times \mathbf{u}') =$$

$$= (-G \times_n M) \times_n ((\mathbf{u}, \mathbf{u}') \times_n \mathbf{u} -_n ((\mathbf{u}, \mathbf{u}) \times_n \mathbf{u}')$$

We consider the equality **(NK6)** as a first approach.

We note:

- probability of equality $\mathbf{a} = ((-G \times_n M) \times_n (\frac{1}{r})^3)\mathbf{r} = ((-G \times_n M) \times_n (\frac{1}{r})^2) \times_n \mathbf{u}$ is less than 1 **(SP4, LS5)**;

- probability of equality

$$((-G \times_n M) \times_n (\frac{1}{r})^2) \times_n \mathbf{u} \times r^2 \times_n (\mathbf{u} \times \mathbf{u}') =$$

$$= (-G \times_n M) \times_n \mathbf{u} \times (\mathbf{u} \times \mathbf{u}')$$

is less than 1 **(LS5)**;

- probability of equality

$$(-G \times_n M) \times_n \mathbf{u} \times (\mathbf{u} \times \mathbf{u}') =$$

$$= (-G \times_n M) \times_n ((\mathbf{u}, \mathbf{u}') \times_n \mathbf{u} -_n ((\mathbf{u}, \mathbf{u}) \times_n \mathbf{u}')$$

is less than 1 **(CP5)**.

(NK6')

$$(r^2 \times_n \mathbf{a}) \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) =$$

$$= -(G \times_n M)) \times_n \mathbf{u} \times ((r^2 \times_n (\mathbf{u} \times \mathbf{u}') +_n r \times_n (\mathbf{u} \times \Delta_6)) +_n \Delta_7) =$$

$$= (-G \times_n M)) \times_n \mathbf{u} \times r^2 \times_n (\mathbf{u} \times \mathbf{u}') +_n$$

$$+_n (-(G \times_n M) \times_n \mathbf{u} \times r \times_n (\mathbf{u} \times \Delta_6)) +_n (-(G \times_n M)) \times_n \mathbf{u} \times \Delta_7) +_n \Delta_8 =$$

$$= (-(G \times_n M) \times_n r^2) \times_n (((\mathbf{u}, \mathbf{u}') \times_n \mathbf{u} -_n ((\mathbf{u}, \mathbf{u}) \times_n \mathbf{u}') +_n$$

$$+_n \Delta_9) +_n (-(G \times_n M) \times_n r \times_n (((\mathbf{u}, \Delta_6) \times_n \mathbf{u} -_n$$

$$-_n ((\mathbf{u}, \mathbf{u}) \times_n \Delta_6) +_n \Delta_{10}) +_n (-(G \times_n M) \times_n \mathbf{u} \times \Delta_7) +_n \Delta_8$$

$\Delta_8, \Delta_9, \Delta_{10}$ are random vectors depend on n.

Since $\mathbf{u}$ is a unit vector, $(\mathbf{u}, \mathbf{u}) = 1$, so

(NK7)

$$\mathbf{a} \times \mathbf{h} = (-G \times_n M) \times_n ((\mathbf{u}, \mathbf{u}') \times_n \mathbf{u} -_n \mathbf{u}')$$

$(\mathbf{u},\mathbf{u})' = \mathbf{0}$ and $(\mathbf{u}',\mathbf{u}) +_n (\mathbf{u},\mathbf{u}') = 0$.

And

(NK8)

$$\mathbf{u} \times \mathbf{h} = (G \times_n M) \times_n \mathbf{u}'$$

We consider the equalities **(NK7, NK8)** as a first approach.

We note:

- probability of equality

$$(\mathbf{u},\mathbf{u})' = (\mathbf{u}',\mathbf{u}) +_n (\mathbf{u},\mathbf{u}')$$

is less than 1 **(D4)**. That means $(\mathbf{u},\mathbf{u}') = \Delta_{11}$, Δ_{11} is a random variable depends on n.

And

(NK8')

$$(r^2 \times_n \mathbf{a}) \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) =$$
$$= ((-(G \times_n M)) \times_n r^2) \times_n ((\Delta_1 1 \times_n \mathbf{u} -_n \mathbf{u}') +_n \Delta_9) +_n$$
$$+_n (-(G \times_n M)) \times_n r \times_n (((\mathbf{u},\Delta_6) \times_n \mathbf{u} -_n \Delta_6) +_n \Delta_{10}) +_n$$
$$+_n (-(G \times_n M)) \times_n \mathbf{u} \times \Delta_7) +_n \Delta_8$$

But $\mathbf{u}$ is a constant vector, so

(NK9)

$$(\mathbf{v} \times \mathbf{h})' = \mathbf{v}' \times \mathbf{h} = \mathbf{a} \times \mathbf{h}$$

So,

(NK9')

$$(\mathbf{v} \times \mathbf{h})' = (G \times_n M) \times_n \mathbf{u}'$$

We consider the equalities **(NK9)** and **(NK9')** as a first approach.

We note:

- probability of equality "$(\mathbf{v} \times \mathbf{h})' = \mathbf{v}' \times \mathbf{h} +_n \mathbf{v} \times \mathbf{h}'$" is less than 1 **(D5)**. That means

(NK9")

$$(r^2 \times_n (\mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3))' =$$
$$= (r^2 \times_n (\mathbf{v}' \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) +_n (\mathbf{v} \times (\Delta_1 +_n \Delta_2))) +_n \Delta_{12} =$$
$$= (r^2 \times_n (\mathbf{a} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) +_n (\mathbf{v} \times (\Delta_1 +_n$$
$$+_n \Delta_2))) +_n \Delta_{12}$$

Δ_{12} is a random vector depends on n.

So,

(NK10)

$$(r^2 \times_n (\mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3))' =$$

$$= ((-(G \times_n M)) \times_n r^2) \times_n ((\Delta_{11} \times_n \mathbf{u} -_n \mathbf{u}') +_n \Delta_9) +_n$$
$$+_n (-(G \times_n M)) \times_n r \times_n (((\mathbf{u}, \Delta_6) \times_n \mathbf{u} -_n \Delta_6) +_n \Delta_{10}) +_n$$
$$+_n (-(G \times_n M)) \times_n \mathbf{u} \times \Delta_7) +_n \Delta_8) +_n ((r^2 \times_n (\mathbf{u} \times (\Delta_1 +_n \Delta_2))$$

(NK10')

$$(r^2 \times_n (\mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3))' =$$
$$= (G \times_n M)) \times_n r^2) \times_n \mathbf{u}' +_n \Delta_{13}$$

where

$$\Delta_{13} = ((-(G \times_n M)) \times_n r^2) \times_n ((\Delta_{11} \times_n \mathbf{u}) +_n \Delta_9) +_n$$
$$+_n (-(G \times_n M)) \times_n r \times_n (((\mathbf{u}, \Delta_6) \times_n \mathbf{u} -_n \Delta_6) +_n \Delta_{10}) +_n$$
$$+_n (-(G \times_n M)) \times_n \mathbf{u} \times \Delta_7) +_n \Delta_8) +_n ((r^2 \times_n (\mathbf{v} \times (\Delta_1 +_n \Delta_2))$$

(NK11)

$$(\mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3))' =$$
$$= (G \times_n M) \times_n \mathbf{u}' +_n \Delta_{14}$$

Δ_{13}, Δ_{14} are random vectors depend on n.

Integrating both sides of equation **(NK9')** yields

(NK12)

$$\mathbf{v} \times \mathbf{h} = (G \times_n M) \times_n \mathbf{u} +_n \mathbf{p}$$

where p is a constant vector. We consider the equality **(NK12)** as a first approach.

We note:

- probability of equality $\mathbf{v} x \mathbf{h} = (G \times_n M) \times_n \mathbf{u} +_n \mathbf{p}$ is less than 1 **(D3)**. That means

(NK12')

$$\mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) =$$
$$= (G \times_n M) \times_n \mathbf{u} +_n \mathbf{p} +_n \Delta_{15} +_n \Delta_{14} \times_n t$$

where p is a constant vector. Δ_{15} is a random vector depends on n.

So,

(NK13)

$$(\mathbf{r}, \mathbf{v} \times \mathbf{h}) = (\mathbf{r}, (G \times_n M) \times_n \mathbf{u} +_n \mathbf{p}) =$$
$$= (G \times_n M) \times_n (\mathbf{r}, \mathbf{u}) +_n (\mathbf{r}, \mathbf{p}) =$$
$$= (G \times_n M) \times_n r +_n (\mathbf{r}, \mathbf{p}) = (G \times_n M) \times_n r +_n r \times_n (\mathbf{u}, \mathbf{p}) =$$
$$= r \times_n (G \times_n M +_n (\mathbf{u}, \mathbf{p}))$$

We consider the equality **(NK13)** as a first approach.

We note:

- probability of equality

$$(\mathbf{r}, (G \times_n M) \times_n \mathbf{u} +_n \mathbf{p}) = (G \times_n M) \times_n (\mathbf{r}, \mathbf{u}) +_n (\mathbf{r}, \mathbf{p})$$

is less than 1 **(SP2, SP3)**;

- probability of equality

$$(G \times_n M) \times_n (\mathbf{r}, \mathbf{u}) +_n (\mathbf{r}, \mathbf{p}) = (G \times_n M) \times_n r +_n (\mathbf{r}, \mathbf{p})$$

is less than 1 **(SP3)**;

- probability of equality

$$(G \times_n M) \times_n r +_n (\mathbf{r}, \mathbf{p}) = (G \times_n M) \times_n r +_n r \times_n (\mathbf{u}, \mathbf{p})$$

is less than 1 **(SP3)**;

- probability of equality

$$(G \times_n M) \times_n r +_n r \times_n (\mathbf{u}, \mathbf{p}) = r \times_n (G \times_n M +_n (\mathbf{u}, \mathbf{p}))$$

is less than 1 **(LS6)**.

That means

(NK13')

$$(\mathbf{r}, \mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) =$$
$$= (\mathbf{r}, ((G \times_n M) \times_n \mathbf{u} +_n \mathbf{p} +_n \Delta_{15} +_n \Delta_{14} \times_n t) =$$
$$= (G \times_n M) \times_n (\mathbf{r}, \mathbf{u}) +_n (\mathbf{r}, \mathbf{p}) +_n (\mathbf{r}, \Delta_{15}) +_n (\mathbf{r}, \Delta_{14} \times_n t) +_n \Delta_{16} =$$
$$= (G \times_n M) \times_n r +_n r \times_n (\mathbf{u}, \mathbf{p}) +_n r \times_n (\mathbf{u}, \Delta_{15}) +_n$$
$$+_n r \times_n (\mathbf{u}, \Delta_{14} \times_n t) +_n \Delta_{16} +_n \Delta_{17}$$

Δ_{16}, Δ_{17} are random variables depend on n.

From another side

(NK14)

$$(\mathbf{r}, \mathbf{v} \times \mathbf{h}) = (\mathbf{r} \times \mathbf{h}, \mathbf{h}) = (\mathbf{h}, \mathbf{h}) = |\mathbf{h}|^2 = h^2$$

where $|\mathbf{h}| = h$. We consider the equality **(NK14)** as a first approach.

We note:

- probability of equality "$(\mathbf{r}, \mathbf{v} \times \mathbf{h}) = (\mathbf{r} \times \mathbf{v}, \mathbf{h})$" is less than 1 **(CP7)**;

- probability of equalities "$(\mathbf{h}, \mathbf{h}) = |\mathbf{h}|^2 = h^2$" is less than 1 **(SP4)**. That means

(NK14')

$$(\mathbf{r}, \mathbf{v} \times (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) =$$

$$= (\mathbf{r} \times \mathbf{v}, \mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) +_n \Delta_{18} =$$

$$= (\mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3, \mathbf{h} +_n (\Delta_1 +_n \Delta_2) \times_n t +_n \Delta_3) +_n \Delta_{18} =$$

$$= h^2 +_n (2 \times_n t) \times_n (\mathbf{h}, \Delta_1 +_n \Delta_2) +_n 2 \times_n (\mathbf{h}, \Delta_3) +_n$$

$$+_n (t \times_n (\Delta_1 +_n \Delta_2), t \times_n (\Delta_1 +_n \Delta_2)) +_n$$

$$+_n 2 \times_n (\Delta_1 +_n \Delta_2, \Delta_3) +_n (\Delta_3, \Delta_3) +_n \Delta_{18} +_n \Delta_{19}$$

Δ_{18}, Δ_{19} are random variables depend on n.

So,

(NK15)

$$h^2 +_n (2 \times_n t) \times_n (\mathbf{h}, \Delta_1 +_n \Delta_2) +_n 2 \times_n (\mathbf{h}, \Delta_3) +_n$$

$$+_n (t \times_n (\Delta_1 +_n \Delta_2), t \times_n (\Delta_1 +_n \Delta_2)) +_n 2 \times_n (\Delta_1 +_n \Delta_2, \Delta_3) +_n$$

$$+_n (\Delta_3, \Delta_3) +_n \Delta_{18} +_n \Delta_{19} =$$

$$= (\mathbf{r}, ((G \times_n M) \times_n \mathbf{u} +_n \mathbf{p} +_n \Delta_{15} +_n \Delta_{14} \times_n t) =$$

$$= (G \times_n M) \times_n (\mathbf{r}, \mathbf{u}) +_n (\mathbf{r}, \mathbf{p}) +_n (\mathbf{r}, \Delta_{15}) +_n$$

$$+_n (\mathbf{r}, \Delta_{14} \times_n t) +_n \Delta_{16} =$$

$$= (G \times_n M)) \times_n r +_n r \times_n (\mathbf{u}, \mathbf{p}) +_n r \times_n (\mathbf{u}, \Delta_{15}) +_n$$

$$+_n r \times_n (\mathbf{u}, \Delta_{14} \times_n t) +_n \Delta_{16} +_n \Delta_{17}$$

And finally **(NK16)**

$$h^2 = r \times_n ((G \times_n M) +_n (\mathbf{u}), \mathbf{p})))$$

We consider the equality **(NK16)** as a first approach.

(NK16')

$$h^2 +_n (2 \times_n t) \times_n (\mathbf{h}), \Delta_1 +_n \Delta_2) +_n 2 \times_n (\mathbf{h}), \Delta_3) +_n$$

$$+_n (t \times_n (\Delta_1 +_n \Delta_2), t \times_n (\Delta_1 +_n \Delta_2)) +_n 2 \times_n (\Delta_1 +_n \Delta_2, \Delta_3) +_n$$

$$(+_n \Delta_3, \Delta_3) +_n \Delta_{18} +_n \Delta_{19} =$$

$$= r \times_n ((((G \times_n M) +_n (\mathbf{u}), \mathbf{p}))) +_n (\mathbf{u}), \Delta_{15})) +_n$$

$$+_n t \times_n (\mathbf{u}), \Delta_{14})) +_n \Delta_{20}$$

Δ_{20} is a random variable depends on n.

(NK16")

$$(h^2 +_n t \times_n (2 \times_n \mathbf{h}), \Delta_1 +_n \Delta_2)) +_n (t \times_n (\Delta_1 +_n \Delta_2), t \times_n (\Delta_1 +_n \Delta_2)) =$$

$$= r \times_n ((G \times_n M +_n (\mathbf{u}), \mathbf{p}))) +_n t \times_n (\mathbf{u}), \Delta_{14})) +_n \Delta_{21}$$

where

$$\Delta_{21} = r \times_n (\mathbf{u}), \Delta_{15}) -_n 2 \times_n (\mathbf{h}), \Delta_3) -_n 2 \times_n (\Delta_1 +_n$$
$$+_n \Delta_2, \Delta_3) -_n (\Delta_3, \Delta_3) -_n \Delta_{18} -_n \Delta_{19} +_n \Delta_{20}$$

Equation **(NK16")** with $\Delta_1 = \mathbf{0}, \Delta_2 = \mathbf{0}, \Delta_{14} = \mathbf{0}, \Delta_{21} = 0$ coincides with equation **(NK16)** and is an ellipse equation in classical linear algebra.

But in Observer's Mathematics we have equation **(NK16")** with $\Delta_1, \Delta_2, \Delta_{14}, \Delta_{21}$, and $\Delta_1, \Delta_2, \Delta_{14}, \Delta_{21}$ are the random variables with multivariable distribution functions depend on n.